On classical super-radiance in Kerr-Newman-anti-de Sitter black holes

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Abstract

We study in detail the modes of a classical scalar field on a Kerr-Newman-anti-de Sitter (KN-AdS) black hole. We construct sets of basis modes appropriate to the two possible boundary conditions ("reflective" and "transparent") at time-like infinity, and consider whether super-radiance is possible. If we employ "reflective" boundary conditions, all modes are non-super-radiant. On the other hand, for "transparent" boundary conditions, the presence of super-radiance depends on our definition of positive frequency. For those KN-AdS black holes having a globally time-like Killing vector, the natural choice of positive frequency leads to no super-radiance. For other KN-AdS black holes, there is a choice of positive frequency which gives no super-radiance, but for other choices there will, in general, be super-radiance.

1 Introduction

Although the subject of quantum field theory on black hole geometries in asymptotically flat space has been widely researched over the past thirty years, there remain some aspects which are not fully understood. Many detailed computations of the expectation values of quantities such as the renormalized expectation value of the stress tensor have been performed on static, spherically symmetric geometries such as Schwarzschild, but the greater complexity of such calculations on rotating black hole space-times has prevented correspondingly exhaustive study. One reason for the greater computational difficulty in rotating black holes is that the metric and other geometric quantities depend on two, rather than one, variable. Another key factor for quantum field theory is the presence of super-radiant modes, as a result of the lack of a globally time-like Killing vector. These need to be dealt with in a subtle manner (see [1, 2] for discussion of this point).

In recent years there has been a great increase in interest in black holes in asymptotically anti-de Sitter space, due to the conjectured correspondence between gravity in the bulk of anti-de Sitter space and conformal field theory on its boundary (the AdS/CFT correspondence - see, for example, [3] for a review and comprehensive list of references). In the light of this, it is natural to instigate a study of quantum field theory on black holes geometries which are asymptotically anti-de Sitter, instead of asymptotically flat. Such an investigation may deepen understanding, not only of the AdS/CFT correspondence itself, but also of the outstanding problems of quantum field theory on asymptotically flat black holes. Quantum field theory on asymptotically anti-de Sitter black holes is also attractive to study since there are black holes in anti-de Sitter space which have a stable Hartle-Hawking state [4], whereas in asymptotically flat space black holes can only be in unstable equilibrium with a heat bath at the Hawking temperature.

There has already been a great deal of work on (both rotating and static) black holes in anti-de Sitter space, primarily the BTZ [5, 6] black holes which live in a three-dimensional universe with a negative cosmological constant. As well as their classical properties, the behaviour of quantum fields on the BTZ black hole has been extensively studied (see, for example, [7, 8, 9, 10]). One advantage of working in three dimensions is that many of the computational aspects are greatly simplified (for example, it is possible to write the Green's function for a quantum field in the Hartle-Hawking state in closed form [11, 12, 13]). However, there has been more recent interest in higher-dimensional black holes in anti-de Sitter space, particularly Kerr-Newman-anti-de Sitter (KN-AdS) black holes in various dimensions [14, 15, 16]. Work to date has concentrated on calculating those quantities of particular interest for the AdS/CFT correspondence, primarily within a classical framework (including the work in Refs. [17, 18, 19]), and there has been little study of conventional quantum field theory on these backgrounds (see, for example, [20] for work to date).

Our purpose in the present article is to lay some foundations for a detailed study of quantum field theory on four-dimensional KN-AdS black holes, by considering the modes of a classical scalar field on this background. In particular, we wish to discover whether the super-radiant modes, which cause so much difficulty in Kerr-Newman space-time, still exist in KN-AdS black holes. If there is an absence of super-radiance for (at least some) KN-AdS black holes, then it may be that quantum field theory in these space-times is, in some ways, simpler than for asymptotically flat Kerr-Newman black holes, and one may hope to tackle such questions as the existence of a Hartle-Hawking state (which does not exist for Kerr-Newman geometries [2, 21, 22]). The present article, however, concentrates on purely classical scalar field modes, a detailed understanding of which is an essential precursor to serious study of quantum field theory effects (for example, for calculating the semi-classical entropy of the black hole [23, 24]). We consider a scalar field as the simplest bosonic field, since for Kerr-Newman black holes in asymptotically flat space, super-radiance occurs for bosonic but not fermionic fields. However, the quantum analogue of super-radiance does occur for both bosonic and fermionic fields [25]. For reviews and references to the original literature on classical super-radiance in Kerr-Newman black holes, the reader is referred to [26] for a scalar fields, and [27] for spin 1/2 fields, electromagnetic and gravitational perturbations.

We begin, in section 2, by reviewing those aspects of the geometry of KN-AdS which are useful for our purposes. One interesting result at this stage is that there are KN-AdS black holes which possess a Killing vector which is time-like everywhere outside the event horizon. This suggests that there may be KN-AdS geometries on which super-radiance is absent. In section 3 we separate the wave equation for scalar fields and study, in particular, the radial equation, by means of a new radial co-ordinate R which turns out to be particularly useful. The next two sections tackle the more subtle issues of boundary conditions and positive frequency, and we construct linearly independent sets of basis modes which will be useful when we return, in a subsequent publication, to the subtle issue of constructing suitable quantum vacuum states. In section 5 we reach our conclusions about the presence (or absence) of super-radiant modes. Due to the time-like nature of infinity $\mathcal{I}$ in KN-AdS, there is a choice of two types of boundary condition at infinity [28]: "reflective" and "transparent". For "reflective" boundary conditions, no particle flux escapes through infinity and super-radiance is absent. This is in agreement with an energy argument given by Hawking and Reall [16]. For "transparent" boundary conditions, whether or not there are super-radiant modes depends on our prescription of positive frequency. For those KN-AdS black holes having a globally time-like Killing vector, the natural definition of positive frequency leads only to non-super-radiant modes. Otherwise, the choice of positive frequency is ambiguous, with the analogue of the conventional choice in Kerr-Newman black holes leading to super-radiant modes (although there is another choice of positive frequency in which all modes are non-super-radiant, whereas in Kerr-Newman super-radiance is inevitable, whatever the choice of positive frequency). Section 6 contains our thoughts on the consequences of these results for quantum field theory.

2 Geometric structure of KN-AdS black holes

In Boyer-Lindquist-like co-ordinates the metric of the KN-AdS black hole takes the form [29]

$$\begin{aligned} ds^2 &= -\frac{\Delta_r}{\rho^2}\left[dt - \frac{a\sin^2\theta}{\Sigma}d\phi\right]^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 \\ &\quad + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left[a\,dt - \frac{r^2+a^2}{\Sigma}d\phi\right]^2, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \rho^2 &= r^2 + a^2\cos^2\theta, \\ \Sigma &= 1 - \frac{a^2}{l^2}, \\ \Delta_r &= \left(r^2+a^2\right)\left(1+\frac{r^2}{l^2}\right) - 2Mr + z^2, \\ \Delta_\theta &= 1 - \frac{a^2}{l^2}\cos^2\theta. \end{aligned}$$

Throughout this paper the metric has signature $(-,+,+,+)$ and we use units in which $G = c = 1$. The parameter a describes the rotation (and vanishes for the Schwarzschild-AdS black hole), M is the mass parameter and z^2 represents the total squared charge (i.e. the sum of the electric charge squared and the magnetic charge squared when both are present). This metric is a solution of the Einstein-Maxwell equations with a negative cosmological constant $\Lambda = -3/l^2$, and is valid only for $a^2 < l^2$. The singular limit in which $a^2 \to l^2$ has been considered by, for example, [15].

For values of the mass parameter M above a critical value M_c, the function Δ_r has two roots: $r = r_+$, which is the location of the outer (event) horizon, and $r = r_-$. Given that Δ_r is a quartic, there is no simple closed form expression for $r_\pm$, although it is straightforward to show that Δ_r has only two positive real roots. If $M < M_c$, then Δ_r has no zeros and there is a naked singularity, whilst for $M = M_c$, we have an extremal black hole. In this paper we shall assume that $M > M_c$ and concentrate on the geometry of the black hole exterior to the event horizon $r = r_+$. The event horizon has angular velocity ω_+ given by

$$\omega_+ = \frac{a\Sigma}{r_+^2 + a^2}.$$

These Boyer-Lindquist-like co-ordinates are the most convenient in which to find the scalar field mode solutions in the next section. It is clear from the form of the metric (1), how the Kerr-Newman geometry arises as the limit $l \to \infty$. However, the use of Boyer-Lindquist-like co-ordinates obscures the structure of the geometry at infinity. At infinity, the metric (1) describes anti-de Sitter space as seen by a rotating observer. This can most readily be seen by making the co-ordinate transformation [30]:

$$T = t, \qquad \Phi = \phi + \frac{a}{l^2}t, \qquad y\cos\Theta = r\cos\theta, \qquad y^2 = \frac{1}{\Sigma}\left[r^2\Delta_\theta + a^2\sin^2\theta\right].$$

A long but straightforward calculation shows that the most compact form for the metric components in these co-ordinates is:

$$\begin{aligned}
g_{TT} &= -1 - \frac{y^2}{l^2} - \frac{\delta\Delta_\theta^2}{\rho^2\Sigma^2}, \\
g_{T\Phi} &= \frac{a\delta\Delta_\theta}{\rho^2\Sigma^2}\sin^2\theta, \\
g_{\Phi\Phi} &= y^2\sin^2\Theta - \frac{a^2\delta}{\rho^2\Sigma^2}\sin^4\theta, \\
g^{yy} &= 1 + \frac{y^2}{l^2} + \frac{\delta\Delta_\theta^2 r^2}{\rho^2\Sigma^2 y^2}, \\
g^{y\Theta} &= -\frac{a^2\delta\Delta_\theta r}{\rho^2\Sigma^{3/2}y^3(r^2+a^2)^{1/2}}\sin\theta\cos\theta, \\
g^{\Theta\Theta} &= \frac{1}{y^2} + \frac{a^4\delta}{\rho^2\Sigma y^4(r^2+a^2)}\sin^2\theta\cos^2\theta,
\end{aligned} \tag{2}$$

where

$$\delta = -2Mr + z^2,$$

and r and θ are functions of y and Θ. In these co-ordinates it becomes apparent that in the case $M = 0$, $z = 0$ (when $\delta = 0$), the metric (1) is simply anti-de Sitter space as seen by a rotating observer, and also that, as $r \to \infty$ (which corresponds to $y \to \infty$) the geometry approaches anti-de Sitter space. The (T, y, Θ, Φ) co-ordinates are therefore more physical and we will refer to them especially for interpreting the modes at infinity. However, due to the greater complexity of the metric (2), it will be easier to perform computations using (t, r, θ, ϕ) co-ordinates and then transform to (T, y, Θ, Φ) co-ordinates. The angular velocity of the event horizon in (T, y, Θ, Φ) co-ordinates is

$$\Omega_+ = \omega_+ + \frac{a}{l^2} = \frac{a(1 + r_+^2 l^{-2})}{r_+^2 + a^2}.$$

In a rotating black hole space-time, there are two surfaces (apart from the event horizon) of importance, especially for quantum field theory. They are the stationary limit surface and the velocity of light surface. The stationary limit surface is the surface where an observer can no longer remain stationary with respect to infinity, but is forced to rotate by the dragging of inertial frames near the event horizon. For the KN-AdS black hole, the stationary limit surface will be the surface on which the Killing vector $\eta = \partial/\partial T$ (which is time-like at infinity) becomes null. Using (T, y, Θ, Φ) co-ordinates, it is straightforward to show that $g_{TT} > 0$ at the event horizon, so that η is space-like close to the event horizon. Therefore, for all KN-AdS black holes, there has to be a stationary limit surface and ergosphere.

The Killing vector generating the event horizon is

$$\zeta = \frac{\partial}{\partial T} + \Omega_+ \frac{\partial}{\partial \Phi},$$

and the velocity of light surface (if it exists) is the surface outside the event horizon on which this vector becomes null. Outside the velocity of light surface, it is impossible for an observer to rigidly rotate with the same angular velocity as the event horizon. As $y \to \infty$,

$$g_{\tau\nu} \zeta^\tau \zeta^\nu = -1 - \frac{y^2}{l^2} + \Omega_+^2 y^2 \sin^2 \Theta + O(y^{-1}).$$

Therefore, if $\Omega_+^2 < l^{-2}$, the Killing vector ζ is time-like at infinity and there is no velocity of light surface [16]. For these black holes possessing a globally time-like Killing vector, as we shall discuss in section 5, there is a natural definition of positive frequency for our classical scalar field modes, and we may anticipate the absence of super-radiance.

Our purpose in the present article is to compare the behaviour of classical scalar field modes in KN-AdS as opposed to ordinary Kerr-Newman black holes. It is therefore useful to sketch the Penrose diagram for the geometry outside the event horizon on the axis of symmetry, which is shown in figure 1 for both the KN-AdS and asymptotically flat Kerr-Newman black holes (cf. Ref. [31]). We also sketch the Penrose diagram for (the covering space of) anti-de Sitter space itself for comparison. In the Penrose diagrams, $\mathcal{H}^\pm$ denote the future and past event horizons, whilst $\mathcal{I}^\pm$ are future and past null

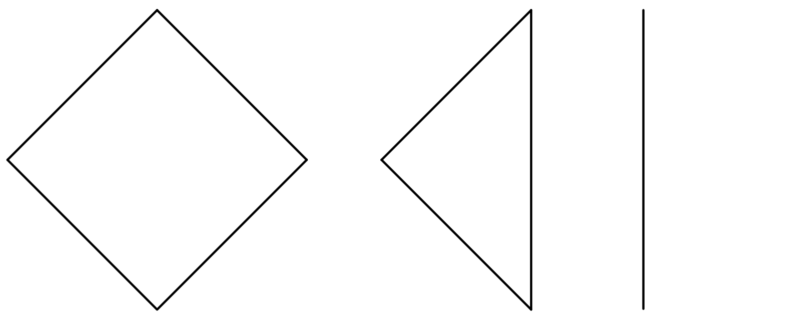

Figure 1: Penrose diagrams for the geometry outside the event horizon of Kerr-Newman and KN-AdS black holes, along the axis of symmetry. We also show the Penrose diagram for (the covering space of) anti-de Sitter space for comparison.

infinity for the Kerr-Newman black hole. For the KN-AdS black hole and anti-de Sitter space, infinity is time-like and denoted simply $\mathcal{I}$. Corresponding Penrose diagrams for BTZ black holes in three-dimensional anti-de Sitter space may be found in [6, 11, 32].

3 Classical scalar field modes

In this section we construct classical field modes for a scalar field with general coupling to the curvature. As is well known, for Kerr-Newman black holes in asymptotically flat space, classical super-radiance effects occur only for bosonic fields [26, 27], although quantum super-radiance occurs for both bosonic and fermionic fields [25]. Since our interest in this paper is classical super-radiance, we therefore need only consider a bosonic field, and we focus on a scalar field for simplicity. It is expected that the scalar field will exhibit all the subtleties associated with both the black hole's rotation and the asymptotic structure of the space-time.

Therefore, we consider a general scalar field Ψ satisfying the equation

$$\left[\nabla_\nu \nabla^\nu - \xi R - \mu^2\right] \Psi = 0, \tag{3}$$

where R is the Ricci scalar of the geometry, ξ is a coupling constant and μ is the mass of the field. At this stage we make no assumptions about the mass μ or the value of the coupling constant ξ. For the KN-AdS geometry, $R = -12/l^2$, so the equation (3) takes the form

$$\left[\nabla_\nu \nabla^\nu - \tilde{\mu}^2\right] \Psi = 0 \tag{4}$$

where

$$\tilde{\mu}^2 = \mu^2 - \frac{12\xi}{l^2}$$

is an effective "mass squared" for the scalar field (although $\tilde{\mu}^2$ need not be positive, for example, for conformally coupled massless fields, $\mu = 0$ and $\xi = 1/6$, so that $\tilde{\mu}^2 = -2/l^2 < 0$). Note that, due to the constancy of the curvature of the geometry, models with different values of ξ and μ can give rise to the same field equation (4). This ambiguity was considered in [33, 34] for scalar fields in super-gravity on anti-de Sitter space.

The wave equation (4) separates in the Boyer-Lindquist-like co-ordinates (t, r, θ, ϕ), so we consider solutions of the form

$$\Psi(t, r, \theta, \phi) = e^{-i\omega t} e^{im\phi} F_{\omega lm}(r) G_{\omega lm}(\theta). \tag{5}$$

For field modes in anti-de Sitter space, regularity of the modes at the origin implies that the frequency ω will take discrete values [28]. For KN-AdS black holes, however, we do not need to impose regularity at the origin and so ω is continuous. The angular quantum number m is required to be an integer, and l is an integer labelling the angular eigenfunctions. When studying quantum field theory on Kerr-Newman black holes, it is conventional at this stage to define "positive frequency" by taking $\omega > 0$ [1]. However, at the moment we shall not fix the sign of ω, and shall postpone a discussion of the appropriate definition of positive frequency until section 5.

The separated modes (5) give the following radial and angular equations:

$$\begin{aligned} 0 &= \Delta_r \partial_r \left(\Delta_r \partial_r F_{\omega lm}(r) \right) \\ &\quad + \left\{ \left[\left(r^2 + a^2 \right) \omega - ma\Sigma \right]^2 - K_{\omega lm} \Delta_r - r^2 \tilde{\mu}^2 \Delta_r \right\} F_{\omega lm}(r), \\ 0 &= \Delta_\theta \sin\theta \partial_\theta \left(\Delta_\theta \sin\theta \partial_\theta G_{\omega lm}(\theta) \right) \\ &\quad + \left\{ K_{\omega lm} \Delta_\theta \sin^2\theta - \left(a\omega \sin^2\theta - m\Sigma \right)^2 - a^2 \tilde{\mu}^2 \Delta_\theta \sin^2\theta \cos^2\theta \right\} G_{\omega lm}, \end{aligned} \tag{6}$$

where $K_{\omega lm}$ is a separation constant. Due to the Δ_θ terms, the angular equation (6) is more complex that that for the usual spheroidal harmonic functions [35]. However, imposing regularity of $G_{\omega lm}(\theta)$ at $\theta = 0$ and π will give a discrete sequence of real eigenvalues $K_{\omega lm}$ which we label by the integer $l = 0, 1, \ldots$ (which, unlike the case in asymptotically flat space, is not necessarily related to the angular quantum number m). We shall take the corresponding eigenfunctions to be real (as is the case for the spheroidal harmonics). Given a lack of a simple formula for the eigenvalues and eigenfunctions for ordinary spheroidal harmonics, we do not anticipate such a formula in the present situation. We will not consider the angular equation further in the present work, and shall focus on the radial equation. However, detailed study of the angular equation will be necessary for any complete computation of quantum field theory expectation values in KN-AdS.

The usual "tortoise" co-ordinate r^* for KN-AdS is defined by the equation

$$\frac{dr^*}{dr} = \frac{r^2 + a^2}{\Delta_r}.$$

If we further define a new radial function $\tilde{F}_{\omega lm}$ by

$$F_{\omega lm}(r) = \left(r^2 + a^2 \right)^{-1/2} \tilde{F}_{\omega lm}(r^*),$$

then $\tilde{F}_{\omega lm}(r^*)$ satisfies the differential equation

$$\partial_{r^*}^2 \tilde{F}_{\omega lm}(r^*) + \tilde{V}_{\omega lm}(r^*) \tilde{F}_{\omega lm}(r^*) = 0,$$

where the potential $\tilde{V}_{\omega lm}(r^*)$ is given by

$$\begin{aligned}\tilde{V}_{\omega lm}(r^*) &= \left[\omega - \frac{ma\Sigma}{r^2+a^2}\right]^2 - \frac{3a^2\Delta_r^2}{(r^2+a^2)^4} - \frac{\Delta_r}{(r^2+a^2)^2}\left(K_{\omega lm} + r^2\tilde{\mu}^2\right) \\ &\quad + \frac{\Delta_r}{(r^2+a^2)^3}\left(2a^2 - 2Mr + 2z^2 - \frac{2r^4}{l^2}\right), \qquad (7)\end{aligned}$$

and r is to be considered as a function of r^*. The variable r^* is particularly useful for studying the structure of the geometry close to the event horizon (and constructing the Penrose diagram given in figure 1). At the event horizon,

$$\tilde{V}_{\omega lm} \sim \tilde{\omega}^2 = (\omega - m\omega_+)^2, \qquad (8)$$

so we get the usual ingoing and outgoing field modes at the event horizon:

$$\tilde{F}_{\omega lm}(r^*) \sim e^{\pm i\tilde{\omega}r^*}. \qquad (9)$$

However, at infinity $r \to \infty$, the "tortoise" co-ordinate r^* approaches a finite value r^*_∞. Further, the potential $\tilde{V}_{\omega lm}$ diverges at infinity unless $\tilde{\mu}^2 = -2/l^2$. Studies of the quasi-normal modes on asymptotically anti-de Sitter black holes (see, for example, [36, 37]) have considered only massless, minimally coupled scalar fields, for which $\tilde{\mu} = 0$. Therefore, in that case, the potential is divergent at infinity and the boundary condition $\tilde{F}_{\omega lm} \to 0$ as $r^* \to r^*_\infty$ was employed. For massless, conformally coupled scalar fields, $\xi = 1/6$ and $\mu = 0$, so that $\tilde{\mu}^2 = -2/l^2$ and the potential (7) remains finite at infinity.

In order to study the wave-like properties of the field modes at infinity, it is helpful to introduce another radial variable, R, which tends to infinity at both the event horizon and infinity, giving an infinite variable range. The simplest such co-ordinate is:

$$R = \log(r - r_+).$$

If we define a new function $f_{\omega lm}(R)$ by

$$F_{\omega lm}(r) = (r - r_+)^{1/2}\,\Delta_r^{-1/2} f_{\omega lm}(R), \qquad (10)$$

then the governing differential equation is now

$$\partial_R^2 f_{\omega lm}(R) + V_{\omega lm}(R) f_{\omega lm}(R) = 0, \qquad (11)$$

where

$$\begin{aligned}V_{\omega lm}(R) &= (r-r_+)^2\,\Delta_r^{-2}\left[\left(r^2+a^2\right)\omega - ma\Sigma\right]^2 - K_{\omega lm}(r-r_+)^2\,\Delta_r^{-1} \\ &\quad -\tilde{\mu}^2 r^2 (r-r_+)^2\,\Delta_r^{-1} - \frac{1}{4} + \frac{1}{4}(r-r_+)^2\,\Delta_r^{-2}\left(\frac{d\Delta_r}{dr}\right)^2 \\ &\quad -\frac{1}{2}(r-r_+)^2\,\Delta_r^{-1}\frac{d^2\Delta_r}{dr^2}.\end{aligned}$$

Near the event horizon, as $R \to -\infty$,

$$V_{\omega lm} \sim \left(\frac{d\Delta_r}{dr} \right)^{-2} \tilde{\omega}^2 = \Omega^2, \tag{12}$$

giving $f_{\omega lm} \sim e^{\pm i\Omega R}$, in agreement with the ingoing and outgoing waves (9). At infinity, $R \to \infty$, the potential $V_{\omega lm}$ remains finite for all values of the coupling constants, which is the main advantage of this choice of co-ordinate. We have

$$V_{\omega lm} \sim -\frac{9}{4} - \tilde{\mu}^2 l^2.$$

The form of the mode solutions at infinity therefore depends on the effective "mass-squared" $\tilde{\mu}^2$ (which, recall, need not be positive). If $\tilde{\mu}^2 < -9/4l^2$ (hereafter referred to as "case 1"), then $V_{\omega lm} \sim +q^2$ at infinity, where q is real, and

$$f_{\omega lm}(R) \sim e^{\pm iqR}.$$

On the other hand, if $\tilde{\mu}^2 > -9/4l^2$, ("case 2") we have $V_{\omega lm} \sim -q^2$ (with q again real), so that

$$f_{\omega lm}(R) \sim e^{\pm qR}.$$

At this stage it is useful to compare the form of our field modes with those studied by other authors on anti-de Sitter space [28]. There has recently been a revival in the study of field modes on anti-de Sitter space due to the AdS/CFT correspondence, according to which field modes in the bulk correspond to either source terms or states in the boundary theory, depending on whether the modes are non-normalizable or normalizable, respectively [8, 38, 39]. In case 1, the radial function $F_{\omega lm}(r)$ behaves like $r^{-3/2}$ at infinity, and all modes are well-behaved at infinity and normalizable. In case 2, the behaviour of the radial function at infinity is:

$$F^{\pm}_{\omega lm} \sim r^{-3/2 \pm q} \tag{13}$$

where

$$q^2 = \frac{9}{4} + \tilde{\mu}^2 l^2 > 0.$$

If $\tilde{\mu} < 0$, then both modes tend to zero at infinity, although only $F^{-}_{\omega lm}$ is normalizable if $q > 1/2$ (which corresponds to $\tilde{\mu}^2 l^2 > -2$). If $\tilde{\mu} \geq 0$, then only $F^{-}_{\omega lm}$ vanishes at infinity, the other mode diverging. It can be checked that the leading behaviour (13) is in agreement with that of the modes in [28] for massless conformally coupled scalar fields (when $\tilde{\mu}^2 l^2 = -2$). For massless, minimally coupled, fields (such as considered in [36]), $\tilde{\mu} = 0$ and only the $F^{-}_{\omega lm}$ mode vanishes at infinity.

4 Boundary conditions and basis modes

In this section we shall construct sets of basis modes for our classical scalar field, subject to appropriate boundary conditions at the event horizon and infinity. By "basis modes", we mean that the general classical solution of the field equation (3) at each point in the geometry can be written as a linear combination of the sets of the modes we shall give below. Given that anti-de Sitter space is non-globally hyperbolic, we do not address the more difficult question of whether providing initial conditions for the field on some surface will uniquely control the evolution of the field through the rest of the space-time. We shall return to this question in a subsequent article when we consider quantum states on KN-AdS.

The analysis of the field modes near the event horizon is straightforward. All modes have the form

$$f_{\omega lm}(R) = A_{\omega lm} e^{i\Omega R} + B_{\omega lm} e^{-i\Omega R}, \tag{14}$$

with the component proportional to $A_{\omega lm}$ corresponding to flux coming up out of the past event horizon $\mathcal{H}^-$ and the component proportional to $B_{\omega lm}$ corresponding to flux going down the future event horizon $\mathcal{H}^+$.

The key factor in any study of fields on anti-de Sitter space is the choice of boundary conditions at infinity. To understand the options, we first write the general solution to the radial equation (11) as $R \to \infty$ as follows:

Case 1: $\tilde{\mu}^2 < -9/4l^2$

$$f_{\omega lm}(R) = C_{\omega lm} e^{iqR} + D_{\omega lm} e^{-iqR}, \tag{15}$$

Case 2: $\tilde{\mu}^2 > -9/4l^2$

$$f_{\omega lm}(R) = \frac{1}{\sqrt{2}} C_{\omega lm} \left(e^{qR} - i e^{-qR} \right) + \frac{1}{\sqrt{2}} D_{\omega lm} \left(e^{qR} + i e^{-qR} \right), \tag{16}$$

where, in both cases, q is real and positive, and $C_{\omega lm}$ and $D_{\omega lm}$ are complex constants.

In order to interpret these modes, we calculate the radial flux at infinity in each case. Since we are considering behaviour at infinity, the appropriate co-ordinates are (T, y, Θ, Φ) and the radial flux is given by

$$j^y = -i g^{yy} \left(\bar{\Psi} \partial_y \Psi - \Psi \partial_y \bar{\Psi} \right),$$

where we use a bar to denote complex conjugate. For our separable wave modes (5), using the chain rule, the reality of the angular functions $G_{\omega lm}(\theta)$

and the form of the radial function $F_{\omega lm}(r)$ (10), the radial flux is now

$$j^y = -\frac{ir(r^2+a^2)}{\rho^2 y}\left(1+\frac{y^2}{l^2}\right)G^2_{\omega lm}(\theta)\Delta_r^{-1}\left(\bar{f}(R)f'(R) - f(R)\bar{f}'(R)\right).$$

Therefore, we need to calculate $\bar{f}(R)f'(R) - f(R)\bar{f}'(R)$ for the two cases given above. The result is identical for the two cases:

$$\bar{f}(R)f'(R) - f(R)\bar{f}'(R) = 2iq\left(|C_{\omega lm}|^2 - |D_{\omega lm}|^2\right).$$

An outgoing flux of particles will correspond to a positive j^y, so, in each case, the modes with coefficients $C_{\omega lm}$ represent outgoing flux, while those with coefficients $D_{\omega lm}$ represent incoming flux. This is in accord with our expectations for those modes which have a wave-like form $e^{\pm iqR}$ at infinity. However, we have shown that linear combinations of those modes with behaviour $e^{\pm qR}$ at infinity also have an incoming/outgoing wave interpretation. This is confirmed by calculating the flux of energy and angular momentum for the modes at infinity. In each case, the $C_{\omega lm}$ parts give an outgoing flux of both energy and angular momentum, while the $D_{\omega lm}$ parts represent incoming energy and angular momentum.

The crucial first step in studying quantum field theory in curved space is the definition of appropriate basis field modes. Our goal in the present article is the construction of such modes. This depends critically on the boundary conditions we impose on the field at infinity. On anti-de Sitter space, a consistent quantum field theory can be formulated for two different boundary conditions on the modes at infinity [28], namely "transparent" and "reflective". Ref. [28] considers only a conformally coupled scalar field, but the concepts apply equally well to the general coupling we are considering here.

"Reflective" boundary conditions correspond to an absence of flux across infinity. In our situation, this means that we must have $|C_{\omega lm}| = |D_{\omega lm}|$ in either (15) or (16), as applicable. Linearly independent sets of basis modes may be constructed by taking, firstly $C_{\omega lm} = D_{\omega lm}$ and secondly $C_{\omega lm} = -D_{\omega lm}$. Thus there are two sets of modes, corresponding to this choice of sign for $C_{\omega lm}$. This is because there are two boundary conditions which lead to zero flux across infinity: Dirichlet (in which the field itself vanishes at the boundary) and Neumann (where the derivative of the field vanishes).

The natural basis modes in this case are therefore defined by, in case 1,

$$\begin{aligned}
f^1_{\omega lm}(R) &\sim \begin{cases} e^{i\Omega R} + B^1_{\omega lm}e^{-i\Omega R} & \text{as } R \to -\infty, \\ \tilde{C}_{\omega lm}\cos(qR) & \text{as } R \to \infty, \end{cases} \\
f^2_{\omega lm}(R) &\sim \begin{cases} e^{i\Omega R} + B^2_{\omega lm}e^{-i\Omega R} & \text{as } R \to -\infty, \\ -\tilde{D}_{\omega lm}\sin(qR) & \text{as } R \to \infty, \end{cases}
\end{aligned}$$

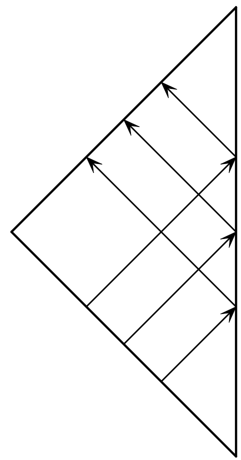

Figure 2: Basis modes on KN-AdS defined by "reflective" boundary conditions at infinity. Flux comes out of the past horizon $\mathcal{H}^-$, is entirely reflected at $\mathcal{I}$ and returns down the future horizon $\mathcal{H}^+$.

and, in case 2,

$$
\begin{aligned}
f^1_{\omega lm}(R) &\sim \begin{cases} e^{i\Omega R} + B^1_{\omega lm} e^{-i\Omega R} & \text{as } R \to -\infty, \\ \frac{1}{\sqrt{2}} \tilde{C}_{\omega lm} e^{qR} & \text{as } R \to \infty, \end{cases} \\
f^2_{\omega lm}(R) &\sim \begin{cases} e^{i\Omega R} + B^2_{\omega lm} e^{-i\Omega R} & \text{as } R \to -\infty, \\ \frac{1}{\sqrt{2}} \tilde{D}_{\omega lm} e^{-qR} & \text{as } R \to \infty. \end{cases}
\end{aligned}
$$

In both cases 1 and 2, the modes $f^1_{\omega lm}$ arise from the choice $C_{\omega lm} = D_{\omega lm}$ (with $\tilde{C}_{\omega lm} = 2C_{\omega lm}$) and the modes $f^2_{\omega lm}$ from $C_{\omega lm} = -D_{\omega lm}$ (with $\tilde{D}_{\omega lm} = -2iC_{\omega lm}$).

In each case, both sets of modes $f^1_{\omega lm}$ and $f^2_{\omega lm}$ represent unit flux coming up out of the past event horizon $\mathcal{H}^-$, being completely reflected at infinity $\mathcal{I}$ and going back down the future event horizon $\mathcal{H}^+$. Note that we do not rule out a change of phase on reflection, so $B^i_{\omega lm}$ does not necessarily equal 1 for $i = 1, 2$. These modes are sketched on the Penrose diagram of KN-AdS in figure 2.

"Transparent" boundary conditions allow flux to escape through time-like infinity. These boundary conditions allow the flow of information through time-like infinity, which needs to be carefully considered if one is interested in quantum states on this background. However, our purposes here are purely classical, and, furthermore, we do not address the difficult issue of how the field evolves in time, and whether such evolution can be fixed by some initial data (given that anti-de Sitter space does not possess a global Cauchy surface). At the same time, these "transparent" boundary conditions are the analogue for asymptotically anti-de Sitter black holes of the usual behaviour of basis field modes on asymptotically flat space-times (where, in general, particles escape to null infinity), and so may be most pertinent if when we subsequently come to consider the construction of appropriate quantum states on KN-AdS. We therefore follow the practice in Kerr space-time [1] and define "in" and "up" modes as follows: in case 1,

$$
\begin{aligned}
f^{in}_{\omega lm} &\sim \begin{cases} B^{in}_{\omega lm} e^{-i\Omega R} & \text{as } R \to -\infty, \\ e^{-iqR} + C^{in}_{\omega lm} e^{iqR} & \text{as } R \to \infty, \end{cases} \\
f^{up}_{\omega lm} &\sim \begin{cases} e^{i\Omega R} + B^{up}_{\omega lm} e^{-i\Omega R} & \text{as } R \to -\infty, \\ C^{up}_{\omega lm} e^{iqR} & \text{as } R \to \infty, \end{cases}
\end{aligned}
$$

and in case 2,

$$
\begin{aligned}
f^{in}_{\omega lm} &\sim \begin{cases} B^{in}_{\omega lm} e^{-i\Omega R} & \text{as } R \to -\infty, \\ \frac{1}{\sqrt{2}}(e^{qR} + ie^{-qR}) + \frac{1}{\sqrt{2}} C^{in}_{\omega lm}(e^{qR} - ie^{-qR}) & \text{as } R \to \infty, \end{cases} \\
f^{up}_{\omega lm} &\sim \begin{cases} e^{i\Omega R} + B^{up}_{\omega lm} e^{-i\Omega R} & \text{as } R \to -\infty, \\ \frac{1}{\sqrt{2}} C^{up}_{\omega lm}(e^{qR} - ie^{-qR}) & \text{as } R \to \infty, \end{cases}
\end{aligned}
$$

In both cases, the $f^{in}_{\omega lm}$ modes represent unit incoming flux from infinity, which is partly reflected back to infinity and partly goes down the future event horizon $\mathcal{H}^+$. The $f^{up}_{\omega lm}$ modes correspond to unit flux coming up from the past event horizon $\mathcal{H}^-$, which is partially reflected back down the future event horizon $\mathcal{H}^+$ and partially transmitted to infinity. These modes are sketched in figure 3 (compare [1, 2] for the corresponding modes in Kerr-Newman space-time).

5 Definition of positive frequency and classical super-radiance

We are now in a position to discuss whether classical scalar field modes in KN-AdS exhibit super-radiance phenomena. For the moment we consider a very general field mode, having behaviour at the event horizon given by (14), and whose form at infinity is given by either (15) or (16), as applicable. If f_1 and f_2 are solutions of the radial equation (11), then the Wronskian

$$
f_1'(R)\bar{f}_2(R) - \bar{f}_2'(R) f_1(R)
$$

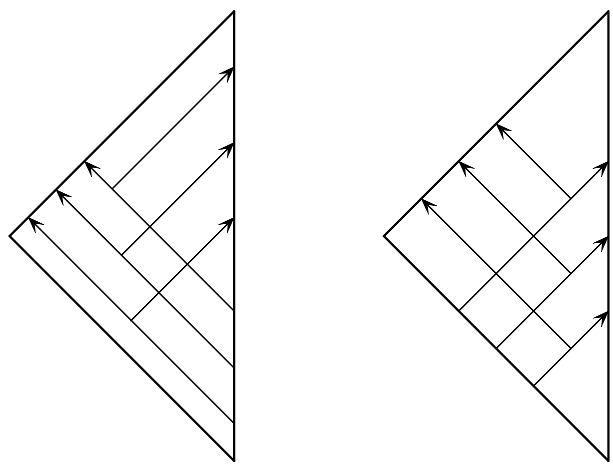

Figure 3: Basis modes on KN-AdS defined by "transparent" boundary conditions at infinity. For the "in" modes, flux comes in from infinity, is partly reflected back to infinity, and partly goes down the future horizon $\mathcal{H}^+$. For the "up" modes, flux comes up from the past horizon $\mathcal{H}^-$, is partly reflected down the future horizon $\mathcal{H}^+$ and partly escapes to infinity.

will be constant. Setting $f_2 = f_1 = f_{\omega lm}$, we arrive at the relation:

$$\Omega \left(|A_{\omega lm}|^2 - |B_{\omega lm}|^2 \right) = q \left(|C_{\omega lm}|^2 - |D_{\omega lm}|^2 \right). \tag{17}$$

We should stress at this stage, that, in contrast to the corresponding equation for scalar field modes in Kerr-Newman space-time [26], here q is by definition always positive and, furthermore, it is a constant for all field modes, depending only on the coupling constants of our original model (see section 3). We now apply equation (17) to each of our sets of basis modes.

Firstly, for modes defined by "reflective" boundary conditions at infinity, $|C_{\omega lm}| = |D_{\omega lm}|$, so the right hand side of (17) vanishes. Thus, for these modes (which were defined with $A_{\omega lm} = 1$), it must be the case that $|B_{\omega lm}| = 1$. This is in agreement with conservation of flux, as there is no flux of particles across $\mathcal{I}$, so the flux emitted from $\mathcal{H}^-$ is entirely reflected back down $\mathcal{H}^+$. Note that it is not necessarily the case that $B_{\omega lm} = 1$ as there may be a phase shift on reflection at infinity. We conclude that, for a field subject to "reflective" boundary conditions at infinity, there is no classical super-radiance. This is in complete accord with the energy argument of Ref. [16], who showed that for matter satisfying the dominant energy condition, and "reflective" boundary conditions, there has to be a net flux of energy down the event horizon.

Secondly, we consider modes defined by "transparent" boundary conditions. For "in" modes, $A^{in}_{\omega lm} = 0$, $D^{in}_{\omega lm} = 1$ and the relation (17) takes the form:

$$-\Omega \left| B^{in}_{\omega lm} \right|^2 = q \left(\left| C^{in}_{\omega lm} \right|^2 - 1 \right).$$

Therefore, for modes with $\Omega < 0$, we have $|C^{in}_{\omega lm}| > 1$, in other words, the flux transmitted out to infinity is greater than the flux incoming from infinity. Similarly, for "up" modes, $D^{up}_{\omega lm} = 0$ and $A^{up}_{\omega lm} = 1$, giving

$$\Omega \left(1 - |B^{up}_{\omega lm}|^2 \right) = q \, |C^{up}_{\omega lm}|^2 .$$

In this case we have $|B^{up}_{\omega lm}| > 1$ for modes with $\Omega < 0$, that is, the flux reflected back down $\mathcal{H}^+$ is greater than that coming up out of $\mathcal{H}^-$. In conclusion, for modes defined with "transparent" boundary conditions at infinity, we have classical super-radiance if $\Omega < 0$.

However, we need at this stage to address the question of whether Ω can be negative in practice. This is closely tied to the definition of positive frequency (which we have side-stepped until now). For rotating black hole geometries, the question of positive frequency is a rather thorny one, and its resolution is one of the reasons that quantum field theory in Kerr-Newman black holes is so much more complex than for Schwarzschild (or Reissner-Nordström) black holes (see, for example, [1] and references therein for some discussion of this problem). The crux of the problem is the lack of a globally time-like Killing vector, which means that a decision has to be made on the location in the geometry of a locally stationary observer who we wish to see only positive frequency modes. In Kerr-Newman, the convention is to take "in" modes to have positive frequency with respect to a locally non-rotating observer (LNRO, also referred to as a zero angular momentum observer (ZAMO) by some authors, for example, [40] or a fiducial observer (FIDO) in other texts, such as [41]) at infinity (the "distant observer" viewpoint [2]), and "up" modes to have positive frequency as measured by an LNRO close to the event horizon (the "near-horizon observer" viewpoint [2]). If we apply this definition of positive frequency in our case, then the time-like Killing vector for an LNRO at infinity is $\partial/\partial T$ rather than $\partial/\partial t$. Therefore we require our "in" modes $\Psi^{in}_{\omega lm}$ to be such that

$$\frac{\partial}{\partial T}\Psi^{in}_{\omega lm} = W\Psi^{in}_{\omega lm}$$

where W should be positive. Using the separation of the field modes (5) we find that

$$W = \omega + \frac{am}{l^2} > 0.$$

Therefore it is possible, with this scheme, to have "in" modes for which $W > 0$ but $\Omega < 0$, and therefore these modes will exhibit super-radiance. For the "up" modes, it is conventional to take $\tilde{\omega}$ (see equation (8)) to be positive. In Kerr-Newman space-time, there are super-radiant "up" modes when $\omega < 0$ but $\tilde{\omega} > 0$. However, in this case there are no super-radiant "up" modes if we define $\tilde{\omega} > 0$, as q is always positive. This reveals how the presence or absence of super-radiant modes is dependent in KN-AdS on the definition of positive frequency, whereas for Kerr-Newman black holes super-radiant modes are inevitable whatever the choice of positive frequency. Indeed, in KN-AdS we could reasonably define positive frequency for "in" modes with respect to an observer near the event horizon rather than an observer at infinity. In this case there would be no super-radiant "in" modes either. However, for general KN-AdS black holes it is by no means obvious from the physical point of view that this is a reasonable strategy.

On the other hand, for a certain class of KN-AdS black holes there is a globally time-like Killing vector. As outlined in section 2, providing the angular velocity of the event horizon in (T, y, Θ, Φ) co-ordinates does not exceed l^{-2}, the Killing vector ζ which is null on the event horizon is time-like everywhere outside the event horizon. In this situation the question of how to define positive frequency has a natural resolution: we require all field modes

$\Psi_{\omega lm}$ to be such that $\zeta\Psi_{\omega lm} = \tilde{W}\Psi_{\omega lm}$ where $\tilde{W} > 0$. Direct calculation reveals that $\tilde{W}$ is in fact $\tilde{\omega}$, so we require that $\tilde{\omega}$ and hence Ω (see equation (12)) be positive for all field modes. Therefore there are no super-radiant modes for these KN-AdS black holes.

We may summarize the results of this section as follows. The presence or absence of super-radiant modes in KN-AdS depends on two factors: the boundary conditions at infinity and our definition of positive frequency modes. There are two possible boundary conditions at infinity. Firstly, "reflective" boundary conditions lead to modes which exhibit no super-radiance, regardless of our definition of positive frequency or the rotation of the black hole. For "transparent" boundary conditions, there is a choice of positive frequency which rules out super-radiant modes. This choice is the natural one for KN-AdS geometries having a globally time-like Killing vector. However, if there is no globally time-like Killing vector, then the choice of positive frequency is ambiguous and super-radiance is possible.

6 Conclusions

The main result of this article is that classical super-radiance is not inevitable for KN-AdS black holes, unlike the situation for rotating black holes in asymptotically flat space. This result is a little surprising given that all KN-AdS black holes have an ergosphere, although energy considerations [16] have suggested that those KN-AdS black holes possessing a globally time-like Killing vector are classically stable.

There are two key properties of classical field modes in KN-AdS as far as super-radiance is concerned: the boundary conditions at infinity and the definition of positive frequency. Both these factors are anticipated to be important in quantum field theory studies. We have shown that using "reflective" boundary conditions (that is, Dirichlet or Neumann boundary conditions) at infinity, for all KN-AdS black hole super-radiance is absent. Using "transparent" boundary conditions, however, the situation depends on the choice of positive frequency. For those KN-AdS black holes having a globally time-like Killing vector, the natural definition of positive frequency implies that there are no super-radiant modes. For other KN-AdS black holes, there is a choice of positive frequency for which super-radiance is absent. This is to be contrasted with the situation for Kerr-Newman black holes in asymptotically flat space, where super-radiance occurs regardless of the definition of positive frequency.

Along the way, we have constructed sets of basis field modes (for both boundary conditions at infinity) which will be useful for studying quantum field theory on KN-AdS. This will be the subject of further work. However, we are encouraged by the results of this paper, since the absence of super-radiance for the cases outlined above removes one of the difficulties of studying quantum field theory on rotating black hole geometries. The next step will be to investigate whether the quantum analogue of super-radiance [25] is present for fields on KN-AdS, given that, for asymptotically flat Kerr-Newman black holes, fermion fields do not classically super-radiate but the quantum spontaneous emission process does occur.

One might hope that quantum field theory will be more straightforward on those KN-AdS black holes having a globally time-like Killing vector, since there is a natural definition of positive frequency. Since these black holes do not have a velocity of light surface, it might even be possible to define a regular Hartle-Hawking state. It is well known that ordinary Kerr-Newman black holes do not possess a Hartle-Hawking state [22], and it is an elementary consequence of the conservation equations that any thermal state which rotates rigidly with the angular velocity of the event horizon must be divergent at the velocity of light surface [21]. This proof breaks down in the absence of a velocity of light surface, although showing that a Hartle-Hawking state can be defined will not be a straightforward problem. In the current paper, we have not addressed the difficult question, fundamental to the construction of quantum states, of how to define appropriate initial data for the field, given that anti-de Sitter space is not globally-hyperbolic. Studies on Kerr black holes [1] have revealed some of the difficulties of trying to define quantum states by the behaviour of the field on a surface which is not a Cauchy surface for the geometry (as happens, for example, when attempting to define the Hartle-Hawking state). These subtleties are anticipated to be more complex when dealing with asymptotically anti-de Sitter black holes, and will be studied in detail in the future.

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